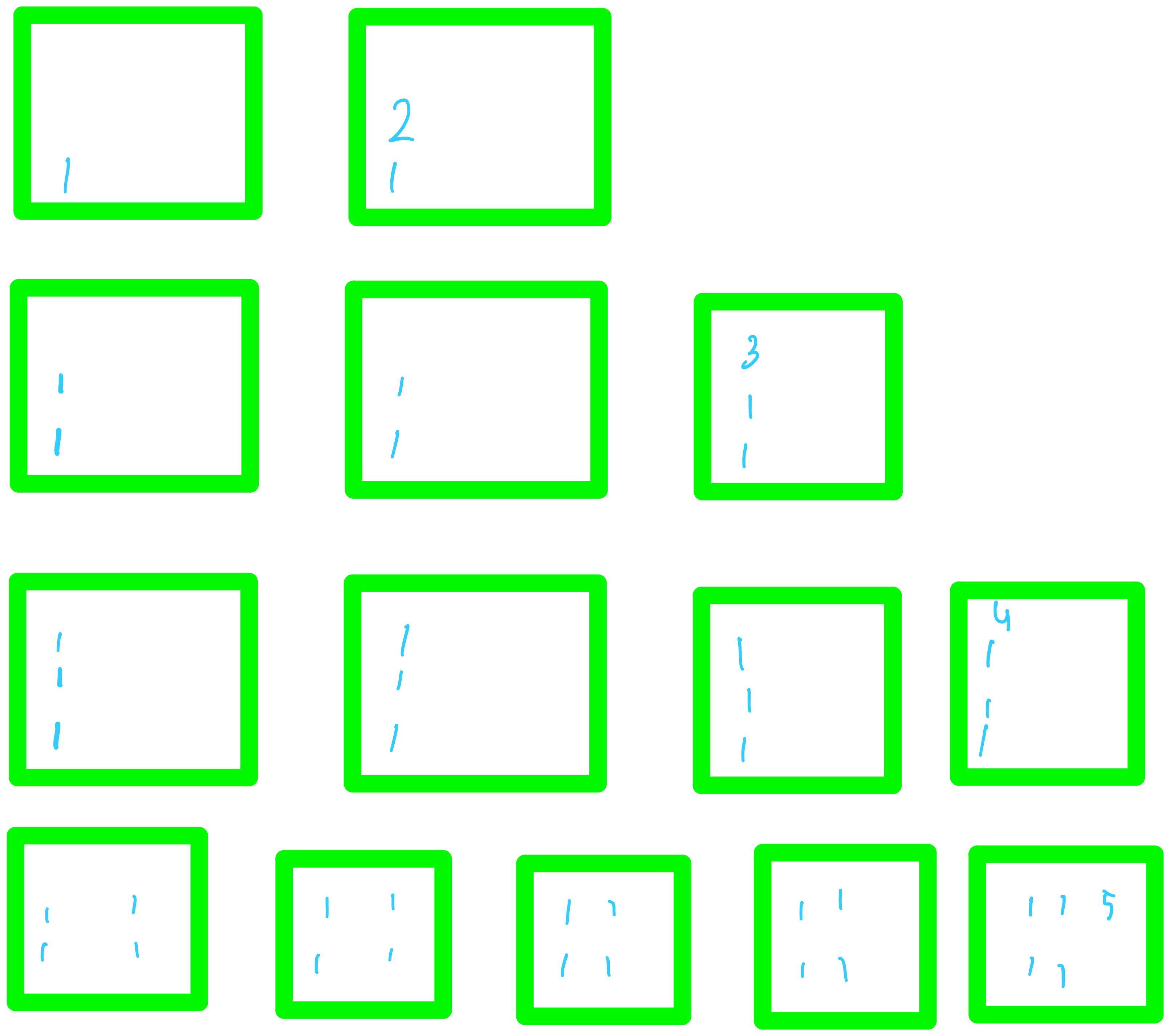
Exercise 1.6

Saturday, 9 September 2023 14:43

$2 - \frac{1}{m}$



$\frac{3}{2}$

$\frac{5}{3}$

$\frac{7}{4}$

$\frac{9}{5}$

Consider the following definition of job times, for jobs numbered from 1 to $n = 2(m-1)+1$:

$$t_i = \begin{cases} m-1 & \text{if } i \le m-1 \\ 1 & \text{if } m-1 < i \le 2(m-1) \\ m & \text{if } i = 2(m-1)+1 \end{cases}$$

In this case, Greedy-Scheduling will always assign the jobs as follows:

1. All of the $m-1$ jobs of size $m-1$ are assigned to their own machine.
2. The $m-1$ jobs of size 1 are the assigned to the remaining machine
3. The last job, of size m, is then assigned to any of the machines.

Whatever machine the last job gets assigned to, the load on that machine will always be $2m-1$.

However, the optimal solution can be obtained as follows:

1. to the first $m-1$ machines, assign a job of size $m-1$ and a job of size 1
   e.g. assign jobs i and $i+m-1$ to machine i, for $1 \le i \le m-1$
2. assign the job of size m to machine m.

This gives a makespan of size m, which must be optimal, since the job of size m will always be assigned to some machine which will have load m.

This implies the approximation ratio of Greedy-Scheduling must be at least

$$\frac{2m-1}{m} = 2 - \frac{1}{m}.$$

□

size of large job = $\frac{\#\text{ small jobs}}{m-1} \cdot$ size of small jobs

assume without loss of generality size of small jobs = 1

small jobs should be divisible by m and $m-1$

Take # small jobs = $m(m-1)$, size of large job = m