Exercise 1.8

Saturday, 9 September 2023 15:22

$m = 2$

$\frac{1}{m}\sum_{1\le j\le n} t_j = \quad \ge t_m$

$\max_{1\le j\le n} t_j = 10 \quad \ge t_m$

$t_m + t_{m+1} = 10 \quad \le 2\cdot t_m$

10

9
9

jobs are {10, 9, 9,

first $m+1$ jobs: equal t_m $\frac{1}{n}\sum \ldots = t_m + \frac{t_m}{n}$

with regards: $t_m + \frac{t_m}{n} + \sum_{j=m+1}^{n} t_j$ should be $\le t_{m+} t_{m+1} \; (\le 2t_m)$

$\frac{t_m}{n} + \sum_{j=m+1}^{n} t_j < t_m$

$t_m + m\sum_{j=m+1}^{n} t_j < m t_m$

$m\sum_{j=m+1}^{n} t_j < (m-1)t_m$

$\sum_{j=m+1}^{n} t_j < \left(1-\frac{1}{m}\right) t_m$

4 OPT = 12
4 4 ALG = 13
5 5

3 OPT = 10
3 3 ALG = 11
5 5

5 OPT = 15
5 5 ALG = 17
7 7

7 OPT = 21
7 7 ALG = 23
9 9

9 27
9 9 29
11 11

11 33
11 = y 11 35
13 = x 13

$\frac{1}{m}\sum_j t_j = 29.5 \quad = x + \frac{3}{2}y$

$\max t_j = 13 \quad = x$ max

solution $= x+2y$ ✗ $\quad \frac{3}{2}\cdot x + \frac{3}{2}y$

will never work

6 OPT = 18
5 5 ALG = 19
9 9

9 OPT = 24
9 9 ALG = 35
17 17

3 3
3 3 3
3 3 3 5 5 5 3
5 5 5 5 5 5 5

dg
go go

x x-1 x-1

solution $= 2x-1$ $= 2x-1$ highest load on machine m (with $t_j \in \mathbb{Z}$)

average $= \frac{1}{2}(x + 2(x-1)) = \frac{1}{2}(3x-2) = \frac{3}{2}x - 1$

$\max = x$

$2x-1 > \frac{3}{2}x \rightarrow \frac{1}{2}x > 1 \rightarrow x > 2$ trivial

$2x-1 > \frac{3}{2}\cdot\left(\frac{3}{2}x-1\right) = \frac{9}{4}x - \frac{3}{2}$

$\frac{1}{2} < \frac{1}{4}x$

$2 < x$

gg
100 100 100

$Load'(M_i^*) + t_{j^*} > \frac{3}{2}\frac{1}{m}\sum_{1\le j\le n} t_j$

$Load'(M_i^*) + t_{j^*} > \frac{3}{2}\max t_j$

If we have a machine with twice the load of all others, we can derive that

note: a machine cannot have more than twice the load of any other machine if all jobs have the same load

$2 > \frac{1}{m}\cdot\frac{3}{2}\cdot(m+1)$

$2 > \frac{3}{2} + \frac{3}{2m}$

$2m > \frac{3}{2}m + \frac{3}{2}$

$\frac{1}{2}m > \frac{3}{2}$

$m > 3$

$m \ge 4$

This works for $m=4$ and the set of jobs $\{100, 100, 100, 100, 100\}$.

Then $\frac{3}{2}\max(\ldots) = \frac{3}{2}\cdot\max\left(\frac{1}{4}\cdot 500, 100\right)$

$= \frac{3}{2}\cdot 125$

$= 187{,}5 < 200$, which is the optimal solution (and hence least possible one obtained by ordered-scheduling)

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Computation exercise...