Exercise 4.6

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1. Probably: take ϵ so small that the size of the solution must be larger than the size of the optimal solution, minus 1.
2. No; the proof depends on the fact that the running time of the FPTAS is polynomial in $\frac{1}{\epsilon}$. If the running time of the PTAS is exponential in $\frac{1}{\epsilon}$, then, since $\epsilon =$, we would have that the running time also becomes exponential in $|V|$, which is perfectly possible, even if Maximum Independent Set is NP-hard and P≠NP.

(i)

Proof: assume $Alg(G, \varepsilon)$ is an FPTAS that computes a $(1-\varepsilon)$-approximation for Maximum Independent Set on a graph G. Then, we can create the following algorithm:

MIS(G):

1. $\varepsilon \leftarrow \frac{1}{n+1}$

2. return $Alg(G, \varepsilon)$

This algorithm provides an exact solution to maximum independent set in polynomial time;

To see why, Note that Alg, as an FPTAS, must provide an answer which is at least $(1-\varepsilon)OPT$ in size. We know that $OPT \leq n$, as no more than all n of the vertices in the graph can be in the maximum independent set. Hence, $(1-\varepsilon)OPT = \mathbf{OPT - \varepsilon \cdot OPT} \leq OPT - \varepsilon n$. Furthermore, we have that the size of a maximum independent set must be an integer; Hence, by setting $\varepsilon = \frac{1}{n+1}$, we obtain that

$\varepsilon n < 1$, and hence, $\varepsilon OPT \leq \varepsilon n < 1$, or: $\varepsilon\, OPT < 1$. Then, $(1-\varepsilon)OPT > OPT - 1$;

this implies that we must have that the size of the solution of the FPTAS is OPT since it must be an integer strictly greater than OPT − 1, and can obviously not be above OPT.

Hence, this algorithm gives an exact solution to the problem of maximum independent set.

Furthermore, since ALG is an FPTAS, the algorithm MIS runs in time polynomial to both n and $\frac{1}{\varepsilon} = n+1$.
Since we have that maximum independent set is now solved in polynomial time, and this problem is NP-hard, we have that P = NP. But assuming P ≠ NP, this gives a contradiction;

hence, it must be the case that our assumption is wrong: we must have that a FPTAS for maximum independent set cannot exist or that P = NP.

(ii) no; a PTAS may still exist. a PTAS may have running time exponential in $\frac{1}{\varepsilon}$, which, for $\varepsilon = \frac{1}{n+1}$, would make the PTAS in MIS (for our choice of ε) run in time exponential in n.
This does not contradict P = NP.