Lecture 3.2

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An improved approximation algorithm for vertex cover works by adding both vertices adjacent to a randomly selected uncovered edge to the vertex cover. Although this will lead to unnecessary vertices being added in (almost?) all cases, this will never lead to more than twice as many vertices being added.

Disjoint edges are edges that do not share an endpoint.

Idea for lower bound: let $E^* \subseteq E$ be a set of pairwise disjoint edges. Then $OPT(G) \geq |E^*|$.
To see why the approximation algorithm is a 2-approximation algorithm, consider that every time a new edge is selected, it is chosen such that it is pairwise disjoint with all the other edges chosen throughout this execution of the algorithm. Furthermore, every such selection leads to 2 nodes being added to the solution size, whereas choosing (in the best case) at least 1 of them would have been necessary. Therefore, the algorithm's solution's size can be at most twice the size of the optimal solution.

It turns out that computing a 1.3606-approximation for Vertex-Cover is NP-hard.