Lecture 3.5

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The 0/1-LP for weighted vertex-cover

Minimize $\sum_{v \in V} weight(v) \cdot x_v$ subject to $x_u + x_v \geq 1$ for each $(u, v) \in E$ and $x_v \in \{0,1\}$ for each $v \in V$.

LP-relaxation

In LP-relaxation, the constraint that x_v should be exactly 0 or 1 is replaced by the constraint stating that $0 \leq x_v \leq 1$.

To determine which vertices should be put in the cover, we work using a rounding scheme. The simplest rounding scheme works by picking a threshold T, then including all elements v with decision variable $x_v \geq T$ in the cover. There is a lemma which states that (for this problem), picking $T = 0.5$ will lead to C being a valid cover. The reason for this is that, for the edge-related constraints to be satisfied, at least one of the decision variables for each edge has to be at least 0.5. Since the vertex for that variable will be included, at least one vertex will be included for each edge. This makes the cover valid.

The relaxed LP-based approximation algorithm for weighted vertex cover runs in polynomial time.