Lecture 3.6

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Any valid solution for the (non-relaxed) 0/1-LP is also a valid solution for the relaxed LP. Thus: the optimal solution for the relaxed LP is at least as good as the optimal solution for the 0/1-LP: $OPT_{relaxed} \leq OPT_{0/1}$.

OPT_{VC} = min weight of vertex cover for the graph G
$OPT_{0/1}$ = value of optimal solution to 0/1-LP.
$OPT_{relaxed}$ = value of optimal solution to relaxed LP = $\sum_{v \in V} weight(v) \cdot x_v$ for the values x_v after solving the relaxed LP

Lemma: $OPT_{VC} = OPT_{0/1} \geq OPT_{relaxed}$

Let G be the input graph and C be the computed cover. Then $WeightedVC - LP - Relaxation(G)$
$= \sum_{v \in C} weight(v)$
$\leq \sum_{v \in C} weight(v) \cdot 2x_v$ since $C = \{v \in V: x_v \geq 0.5\}$ (which implies $2x_v \geq 1$)
$= 2 \sum_{v \in C} weight(v) \cdot x_v$
$\leq 2 \sum_{v \in V} weight(v) \cdot x_v$ since $x_v \geq 0$ for all $v \in V$
$= 2 \cdot OPT_{relaxed}$
$\leq 2 \cdot OPT_{VC}$
This *(combined with the fact that the algorithm returns a valid cover in polynomial time)* proves WeightedVC-LP-Relaxation is a 2-approximation algorithm for weighted vertex cover.

Problems can be solved using LP-relaxation by taking the following steps:

1. Formulate the problem as a 0/1-LP;
    a. Define suitable decision variables;
    b. Define cost function and constraints that model the problem, including 0/1-constraints;
2. Relax the 0/1-constraints;
3. Solve the relaxed LP and round variables;
4. Prove a bound on the approximation ratio using the solution to the relaxed LP as a lower bound.