

# Lecture 4.1

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A polynomial-time approximation scheme is an algorithm which takes as input:

- A problem instance  $I$ ;
- A parameter  $\epsilon > 0$ , which determines how close the solution computed by the algorithm will be to an optimal solution.

More formally,  $Alg$  is a polynomial-time approximation scheme (PTAS) for a minimization problem if

- $Alg(I, \epsilon) \leq (1 + \epsilon) \cdot OPT(I)$  for all inputs  $I, \epsilon$ ;
- The running time of  $Alg$  is polynomial in the size of the input instance  $I$ .

The running time (may) also depend(s) on  $\epsilon$  now. It should be noted that the dependency on  $n$  should always be polynomial. However, the dependency on  $\epsilon$  may be exponential in  $\frac{1}{\epsilon}$ . If the PTAS is polynomial in  $\frac{1}{\epsilon}$ , it is known as a fully polynomial-time approximation scheme, or FPTAS.

For a maximization problem, we have that  $Alg$  is a polynomial-time approximation scheme (PTAS) if

- $Alg(I, \epsilon) \geq (1 - \epsilon) \cdot OPT(I)$  for all inputs  $I, \epsilon$ ;
- The running time of  $Alg$  is polynomial in the size of the input instance  $I$ .