Exercise 7.6

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1. Turn the undirected graph $G = (V, E)$ into a directed graph $G^* = (V, E^*)$, where the edges are obtained by directing every edge from the node with smaller index to the node with higher index: $E^* = \{(v_i, v_j) \in E | i < j\}$. Note that this new graph has its nodes stored in topological order by construction. Furthermore, note that this graph G^* can be obtained by simply ignoring any edges which are not part of the set E^* in the adjacency-list representation of the graph, and hence, no additional I/Os are necessary at this point to use G^*.
2. Now, define a function f on the nodes as follows. (In this application, we do not need to introduce labels $\lambda(v_i)$ to define f.)
   a. If $|N_{in}(v_i)| = 0$, then $f(v_i) = 1$.
   b. If $|N_{in}(v_i)| > 0$, then $f(v_i) = \min_{c \in \{j \in \mathbb{N} | 1 \leq j \leq d_{max}+1\} \setminus \{f(v_{in}) | v_{in} \in N_{in}(v_i)\}} c$.
3. Now, let the f-value of each node denote the color assigned to that node. Then, this algorithm will assign at most $d_{max} + 1$ different colors. To see why, consider that, for each node in the graph, the algorithm either assigns color 1 (if the node has no incoming edges) or the lowest-numbered color which is not already in used by one of the neighbors of the node. Since each node has at most d_{max} incoming edges, this means that no node will be assigned a color higher than $d_{max} + 1$ (given that at least one of the $d_{max} + 1$ colors must be unused by at least one of the neighbors).
4. Note that f is a local function, and that each $f(v_i)$ can be computed in $O(SORT(1 + |N_{in}(v_i)|))$ I/Os from the f-values of its in-neighbors. Then, it follows from theorem 7.2 that the total number of I/Os performed by the algorithm is $O(SORT(|V| + |E|))$.

   explain how, e.g. sort set to be removed, then find minimal missing element