Lecture 12.1

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Expectation values are linear: $E[X+Y]=E[X]+E[Y]$ and $E[cX]=c\cdot E[x]$.
Note that $E[X\cdot Y]=E[X]\cdot E[Y]$ only holds if X and Y are independent.

Indicator random variables are 1 if an event occurs, and 0 if an event does not occur.

Some useful results

If an experiment is successful with probability p, and the experiment is repeated until success, then $E[\#\text{trials until success}]=\frac{1}{p}$.

Markov inequality: for non-negative random variable X and any $t>0$, we have $\Pr[X>t\cdot E[X]]<\frac{1}{\mathrm{t}}$.

Take a set $X_1,\dots,X_k$ to be independent indicator random variables, interpreted such that $X_i=1$ if the ith trial is unsuccessful. Let $X=\sum_{i=1}^{k}X_i$, which can be interpreted as the number of unsuccessful trials. Then, $E[X]$ is the expected number of unsuccessful trials. The Chernoff bound for Poisson trials then states that, for any $\delta>0$, we have that $\Pr[X>(1+\delta)E[X]]<\left(\frac{e^{\delta}}{(1+\delta)^{(1+\delta)}}\right)^{E[X]}$.